Pertemuan ke 13

Resonansi

Suatu rangkaian dikatakan beresonansi ketika tegangan terpasang V dan arus yang dihasilkan I dalam kondisi satu phasa.

Misalkan:

$$V = A\angle\alpha^0$$

$$I = B\angle\beta^0$$

Dalam kondisi satu phasa: $\alpha^0 = \beta^0$, sehingga:

$$Z = \frac{V}{I} = \frac{A\angle\alpha^0}{B\angle\beta^0} = \frac{A}{B}\angle(\alpha^0 - \beta^0) = \frac{A}{B}\angle 0^0 = \frac{A}{B}$$

Terlihat bahwa ketika V dan I satu phasa, impedansi yang dihasilkan seluruhnya komponen riil atau impedansi kompleks haya terdiri dari komponen resistor murni®. Dengan kata lain konsep resonansi adalah menghilangkan komponen imaginer/reaktansi saling meniadakan.

Resonansi Seri:

Impedansi total:

$$Z_{tot} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Saat resonansi:

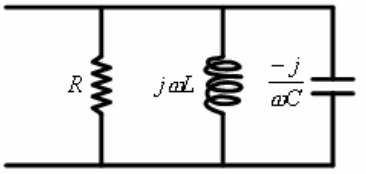
$$\omega L - \frac{1}{\omega C} = 0 \rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi Z minimum, sehingga arusnya maksimum.

Resonansi Paralel



Admitansi total:

$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{-j/\omega C} = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Saat resonansi:

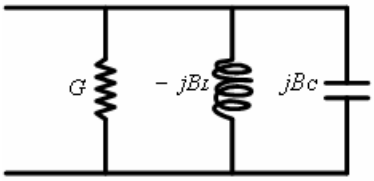
$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi Z maksimum, sehingga arusnya minimum.

Gambar tersebut dapat diganti notasinya:



Admitansi total:

$$Y = G + jB_c - jB_L$$

$$Y = G + j(\omega C - \frac{1}{\omega L})$$

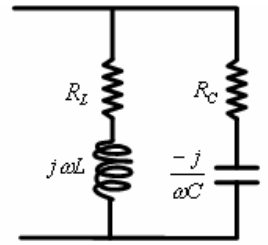
Saat resonansi:

$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Resonansi parallel 2 cabang



$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{1}{R_L + j\omega L} + \frac{1}{R_c - \frac{j}{\omega C}}$$

$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right) + \frac{1}{R_c - \frac{j}{\omega C}}\left(\frac{R_c + \frac{j}{\omega C}}{R_c + \frac{j}{\omega C}}\right)$$

$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{R_L - j\omega L}{{R_L}^2 - (j\omega L)^2} + \frac{R_c + \frac{j}{\omega C}}{{R_c}^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\frac{1}{\boldsymbol{Z}_{tot}} = \frac{R_L}{{R_L}^2 + (\omega L)^2} + \frac{R_C}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} + j\left[\frac{\frac{1}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{{R_L}^2 + (\omega L)^2}\right]$$

Saat resonansi:

$$\frac{\frac{1}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{{R_L}^2 + (\omega L)^2} = 0$$

$${R_L}^2 + (\omega L)^2 = \omega^2 LC\left[{R_C}^2 + \left[\frac{1}{\omega C}\right]^2\right]$$

$${R_L}^2 + \omega^2 L^2 = \omega^2 LC{R_C}^2 + \frac{L}{C}$$

$$\omega^2 LC{R_C}^2 - \omega^2 L^2 = {R_L}^2 - \frac{L}{C}$$

$$\omega^2 LC\left({R_C}^2 - \frac{L}{C}\right) = {R_L}^2 - \frac{L}{C}$$

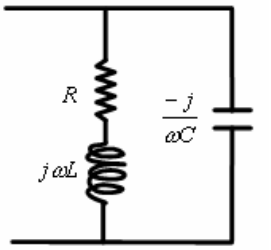
$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{{R_L}^2 - \frac{L}{C}}{{R_C}^2 - \frac{L}{C}}}$$

Perlu diingat bahwa: $\sqrt{\frac{{R_L}^2 - \frac{L}{C}}{{R_C}^2 - \frac{L}{C}}}$ harus positif real sehingga syarat:

${R_L}^2 > \frac{L}{C}$ dan ${R_C}^2 > \frac{L}{C}$ atau ${R_L}^2 < \frac{L}{C}$ dan ${R_C}^2 < \frac{L}{C}$

Ketika nilai ${R_L}^2 = {R_C}^2 = \frac{L}{C}$, maka rangkaian beresonansi untuk semua frekuensi.

Resonansi kombinasi I



$$Z_1 = R + j\omega L$$

$$Z_2 = \frac{1}{j\omega C}$$

$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R + j\omega L} + j\omega C$$

$$\frac{1}{Z_{tot}} = j\omega C + \frac{1}{R + j\omega L}\left(\frac{R - j\omega L}{R - j\omega L}\right)$$

$$\frac{1}{Z_{tot}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

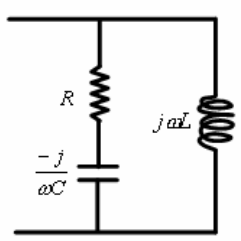
$$\frac{1}{Z_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

Saat resonansi: $\omega_c = \frac{\omega_L}{R^2 + \omega^2 L^2}$, sehingga:

$$R^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \rightarrow \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \frac{1}{LC}\left(1 - \frac{R^2 C}{L^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2 C}{L^2}\right)}$$

Resonansi kombinasi 2



$$Z_1 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$Z_2 = j\omega L$$

$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} = \frac{1}{R - \frac{j}{\omega C}} + \frac{1}{j\omega L} = \frac{1}{R - \frac{j}{\omega C}} - \frac{j}{\omega L}$$

$$\frac{1}{Z_{tot}} = \frac{1}{R - \frac{j}{\omega C}}\left(\frac{R + \frac{j}{\omega C}}{R + \frac{j}{\omega C}}\right) - \frac{j}{\omega L} = \frac{R + \frac{j}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} - \frac{j}{\omega L}$$

$$\frac{1}{Z_{tot}} = \frac{R}{R^2 + \frac{1}{\omega^2 C^2}} + j\left(\frac{\frac{1}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} - \frac{1}{\omega L}\right)$$

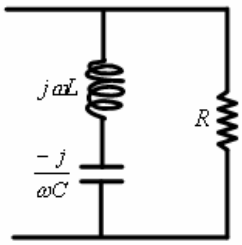
Saat resonansi: $\frac{\frac{1}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}} - \frac{1}{\omega L}$, sehingga:

$$R^2 + \frac{1}{\omega^2 C^2} = \frac{L}{C} \rightarrow \frac{1}{\omega^2 C^2} = \frac{L}{C} - R^2 \rightarrow \omega^2 C^2 = \frac{1}{\frac{L}{C} - R^2} \rightarrow \omega^2 = \frac{1}{C^2\left(\frac{L}{C} - R^2\right)}$$

$$\omega^2 = \frac{1}{LC - C^2R^2} = \frac{1}{LC\left(1 - \frac{L}{C}\right)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(\frac{1}{1 - \frac{CR^2}{L}}\right)}$$

Reonansi kombinasi 3



$$Z_1 = j\omega L + \frac{1}{j\omega C}$$

$$Z_2 = R$$

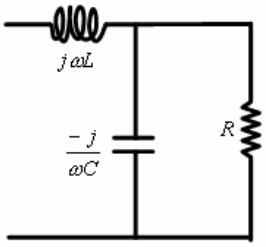
$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{j\omega L + \frac{1}{j\omega C}} + \frac{1}{R} = \frac{1}{R} - \frac{j\omega C}{1 - \omega^2 LC}$$

Saat resonansi:

$$\frac{\omega C}{1 - \omega^2 LC} = 0$$

$$f_0 = 0$$

Resonansi Kombinasi 4



$$\frac{1}{Z_1} = \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R} + j\omega C \rightarrow Z_1 = \frac{1}{\frac{1}{R} + j\omega C}$$

$$Z_2 = j\omega L$$

$$Z_{tot} = Z_1 + Z_2 = \frac{1}{\frac{1}{R} + j\omega C} + j\omega L$$

$$Z_{tot} = j\omega L + \frac{1}{\frac{1}{R} + j\omega C}\left(\frac{\frac{1}{R} - j\omega C}{\frac{1}{R} - j\omega C}\right) = j\omega L + \frac{\frac{1}{R} - j\omega C}{\frac{1}{R^2} + \omega^2 C^2}$$

$$Z_{tot} = \frac{\frac{1}{R}}{\frac{1}{R^2} + \omega^2 C^2} + j\left(\omega L - \frac{\omega C}{\frac{1}{R^2} + \omega^2 C^2}\right)$$

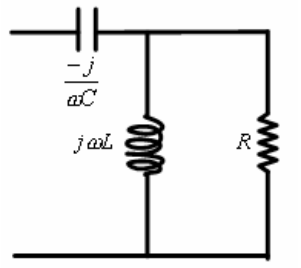
Saat resonansi: $= \frac{\omega C}{\frac{1}{R^2} + \omega^2 C^2}$, sehingga:

$$\frac{1}{R^2} + \omega^2 C^2 = \frac{C}{L} \rightarrow \omega^2 C^2 = \frac{C}{L} - \frac{1}{R^2}$$

$$\omega^2 = \frac{1}{C^2}\left(\frac{C}{L} - \frac{1}{R^2}\right) = \frac{1}{LC} - \frac{1}{C^2R^2} = \frac{1}{LC}\left(1 - \frac{L}{CR^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{L}{CR^2}}$$

Resonansi kombinasi 5



$$\frac{1}{Z_1} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - \frac{j}{\omega L} \rightarrow Z_1 = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}}$$

$$\frac{1}{Z_2} = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$Z_{tot} = Z_1 + Z_2 = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}} - \frac{j}{\omega C} = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}}\left(\frac{\frac{1}{R} + \frac{j}{\omega L}}{\frac{1}{R} + \frac{j}{\omega L}}\right) - \frac{j}{\omega C}$$

$$Z_{tot} = \frac{\frac{1}{R} + \frac{j}{\omega L}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} - \frac{j}{\omega C} = \frac{\frac{1}{R}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} + j\left[\frac{\frac{1}{\omega L}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} - \frac{1}{\omega C}\right]$$

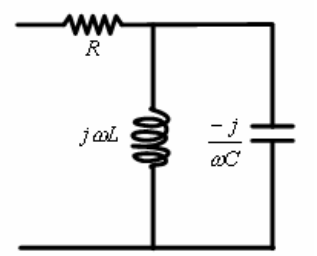
Saat resonansi: $\frac{\frac{1}{\omega L}}{\frac{1}{R^2}+\frac{1}{\omega^2 L^2}} = \frac{1}{\omega C}$, sehingga:

$$\frac{1}{R^2} + \frac{1}{\omega^2 L^2} = \frac{C}{L} \rightarrow \frac{1}{\omega^2 L^2} = \frac{C}{L} - \frac{1}{R^2} \rightarrow \omega^2 L^2 = \frac{1}{\frac{C}{L} - \frac{1}{R^2}}$$

$$\omega^2 = \frac{1}{L^2\left(\frac{C}{L} - \frac{1}{R^2}\right)} = \frac{1}{LC\left(1 - \frac{1}{CR^2}\right)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \frac{L}{CR^2}}}$$

Resonansi kombinasi 6



$$\frac{1}{Z_1} = \frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega L}} = \frac{1}{j\omega L} + j\omega C \rightarrow Z_1 = \frac{1}{\frac{1}{j\omega L} + j\omega C}$$

$$Z_2 = R$$

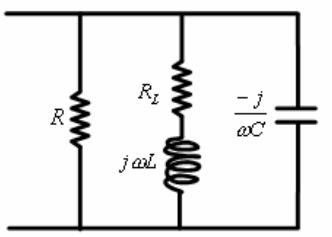
$$Z_{tot} = Z_1 + Z_2 = \frac{1}{\frac{1}{j\omega L} + j\omega C} + R = R + \frac{j\omega L}{1 - \omega^2 LC}$$

Saat resonansi:

$$\frac{\omega L}{1 - \omega^2 LC} = 0$$

$$f_0 = 0$$

Resonansi parallel 3 cabang



$$Z_1 = R_L + j\omega L$$

$$Z_2 = R$$

$$Z_3 = \frac{1}{j\omega C}$$

$$\frac{1}{Z_{tot}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{R_L + j\omega L} + \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right)$$

$$\frac{1}{Z_{tot}} = \frac{1}{R} + j\omega L + \frac{R_L - j\omega L}{{R_L}^2 - \omega^2 L^2} = \frac{1}{R} + \frac{R_L}{{R_L}^2 - \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{{R_L}^2 - \omega^2 L^2}\right)$$

Saat resonansi: $\omega C = \frac{\omega L}{{R_L}^2 - \omega^2 L^2}$, sehingga:

$$R_L{}^2 - \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R_L{}^2$$

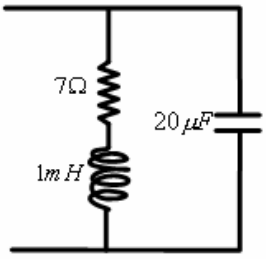
$$\omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R_L{}^2\right) = \frac{1}{LC} - \frac{R_L{}^2}{L^2} = \frac{1}{LC}\left(1 - \frac{CR_L{}^2}{L}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR_L{}^2}{L}}$$

Contoh latihan:

1. Suatu rangkaian seri RLC dengan $R = 50\Omega, L = 0{,}05H, C = 20\mu F$ terpasang pada $V = 100\angle 0^0$ dengan frekuensi variabel. Pada frekuensi berapa tegangan inductor mencapai maksimum? Berapakah tegangan inductor tersebut?
   Jawaban:
   Tegangan induktor maksimum jika arus maksimum arus jika z minimum z minimum terjadi saat resonansi.
   $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0{,}05.20.10^{-6}}} = 159{,}1H_z$$
   $$Z_{resonansi} = R \rightarrow i_{maks} = \frac{V}{Z_{ref}} = \frac{100\angle 0^0}{50} = 2\angle 0^0$$
   $$V_L = i_{maks}X_L = i_{maks}\, j\omega L = 2\angle 0^0 2\pi f L\angle 90^0 = 2\angle 0^0 2\pi 159{,}1.0{,}05\angle 90^0 = 100\angle 90^0$$

2. Pada saat terjadi resonansi tegangan terpasang pada rangkaian seri RLC adalah $v = 70{,}7\sin(500t + 30^0)\, V$ menghasilkan arus sebesar $i = 2{,}83\sin(500t + 30^0)\, A$, jika $L = 0{,}5H$. Tentukan nilai R dan C!
   Jawaban:
   $$Z = \frac{V}{I} = \frac{70{,}7\angle 30^0}{2{,}83\angle 30^0} = 25 \rightarrow R = 25\Omega$$
   $$f_0 = \frac{1}{2\pi\sqrt{30^0}} \rightarrow \omega^2 = \frac{1}{LC}$$
   $$C = \frac{1}{\omega^2 L} = \frac{1}{500^2.0{,}5} = 8\mu F$$
3. Tentukan frekuensi pada gambar berikut:



   Jawaban:

$$\frac{1}{Z_{tot}} = j\omega C + \frac{1}{R + j\omega L}\left(\frac{R - j\omega L}{R - j\omega L}\right)$$
$$\frac{1}{Z_{tot}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$
$$\frac{1}{Z_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

Saat resonansi: $\omega_C = \frac{\omega_L}{R^2+\omega^2 L^2}$, sehingga:

$$R^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \rightarrow \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \frac{1}{LC}\left(1 - \frac{R^2 C}{L^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2 C}{L^2}\right)} = \frac{1}{2\pi\sqrt{10^{-3}20.10^{-6}}}\sqrt{\left(1 - \frac{7^2.20.10^{-6}}{10^{-3}}\right)} = 159{,}2Hz$$

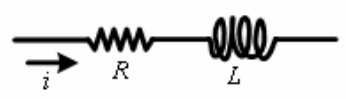
Faktor Kualitas (Q)

Definisi (dasar) dari Q:

$$Q = 2\pi \frac{energi\ maksimum\ yang\ disimpan}{energi\ yang\ disipasikan\ tiap\ getaran\ "percycle"}$$

Factor kualitas merupakan ukuraan selektivitas rangkaian resonator dimana rangkaian resonator merupakan rangkaian filter BPF dengan lebar pita/bandwidth sempit. Semakin besar nilai Q maka semakin sempit lebar pita/bandwidth.

Pada komponen RL



Misalkan: $i = I_{in} sin\omega t$

Pada L:

$$V_L(t) = L\frac{di}{dt} = I_{in}\omega L\cos\omega t$$

Energi:

$$W_L(t) = \int_0^t P_L(t)dt = \int_0^t V_L(t)i(t)dt$$

$$W_L(t) = \int_0^t {I_{in}}^2 \omega L \sin\omega t\cos\omega t dt = \int_0^t {I_{in}}^2 \omega L\frac{sin2\omega t}{2}dt = \frac{{I_{in}}^2\omega L}{2}\int_0^t sin2\,\omega t dt = \frac{1}{2}{I_{in}}^2 L sin^2\omega t$$

Maksimum energy yang disimpan: $W_L \max = \frac{1}{2}L{I_{in}}^2$

Pada R:

Energi:

$$W_R(t) = \int_0^t P_R(t)dt = \int_0^t V_R(t)i(t)dt = \int_0^t RI_m{}^2 sin^2\omega t dt = \int_0^t RI_m{}^2 \frac{(1-cos2\omega t)}{2} dt$$

$$W_R(t) = \frac{RI_m{}^2}{2}\int_0^t (1-\cos 2\omega t)dt = \frac{RI_m{}^2}{2}\left(t - \frac{1}{2\omega} sin2\omega t\right) \rightarrow T = \frac{1}{f} = \left(t - \frac{1}{2\omega} sin2\omega t\right)$$

Energi yang didisipasikan per cycle: $\frac{1}{2}RI_m{}^2\frac{1}{f}$, sehingga:

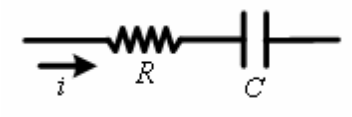
$$Q_L = 2\pi \frac{energi\ maksimum\ yang\ disimpan}{energi\ yang\ disipasikan\ tiap\ getaran/"per\ cycle"}$$

$$Q_L = 2\pi \frac{\frac{1}{2}LI_m{}^2}{\frac{1}{2}RI_m{}^2\frac{1}{f}} = 2\pi f\frac{L}{R} = \frac{\omega L}{R}$$

Jadi factor kualitas untuk rangkaian seri RL:

$$Q_L = \frac{\omega L}{R}$$

Pada komponen RC



Misalkan: $V_C = V_m sin\ \omega t$

Pada C:

$$I_c(t) = C\frac{dV_c}{dt} = CV_m\omega\cos\omega t$$

Energi:

$$W_c(t) = \int_0^t P_c(t)dt = \int_0^t V_c(t)i_c(t)dt$$

$$W_c(t) = \int_0^t V_m{}^2\omega C\sin\omega t\cos\omega t dt = \int_0^t V_m{}^2\omega C\frac{sin2\omega t}{2}dt = \frac{V_m{}^2\omega C}{2}\int_0^t \sin 2\omega t dt = \frac{1}{2}V_m{}^2 C sin^2\omega t$$

Maksimum energi yang disimpan: $W_c max = \frac{1}{2} {V_m}^2$

Pada R:

Energi:

$$W_R(t) = \int_0^t P_R(t)dt$$

$$= \int_0^t V_R(t) I_C(t)dt = \int_0^t R{i_c}^2(t)dt = \int_0^t R(CV_m\omega)^2 cos^2\omega t dt = R(CV_m\omega)^2 \int_0^t cos^2\omega t dt$$

$$W_R(t) = R(CV_m\omega)^2 \int_0^t \frac{cos2\omega t - 1}{2} dt = \frac{R(CV_m\omega)^2}{2}\left(\frac{1}{2\omega} sin2\omega t - t\right) \rightarrow T = \frac{1}{f} = \frac{1}{2\omega} sin2\omega t - t$$

Energi yang didisipasikan per cycle: $\frac{1}{2} R(CV_m\omega)^2 \frac{1}{f}$, sehingga:

$$Q_c = 2\pi \frac{energi\ maksimum\ yang\ disimpan}{energi\ yang\ disipasikan\ tiap\ getaran/"per\ cycle"}$$

$$Q_c = 2\pi \frac{\frac{1}{2} {CV_m}^2}{\frac{1}{2} R(CV_m\omega)^2 \frac{1}{f}} = 2\pi f \frac{1}{\omega^2 RC} = \frac{1}{\omega RC}$$

Jadi factor kualitas untuk rangkaian seri RC:

$$Q_c = \frac{1}{\omega RC}$$

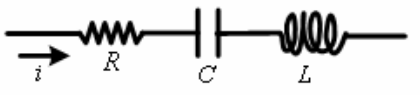
Dapat diambil kesimpulan bahwa factor kualitas (Q) untuk rangkaian seri:

$$Q_s = \frac{X_s}{R_s}$$

Untuk rangkaian seri RL: $Q_s = \frac{\omega_0 L}{R}$

Untuk rangkaian seri RC: $Q_s = \frac{1}{\omega_0 CR}$

Pada komponen RLC



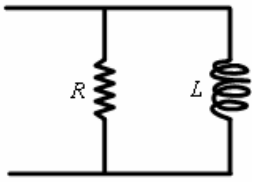
Pada saat terjadi resonansi:

$$\omega^2 = \frac{1}{LC} \rightarrow \omega L = \frac{1}{\omega C}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

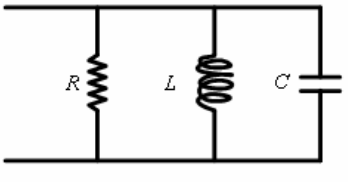
Factor kualitas atau Q pada rangkaian parallel sedikit berbeda dengan Q pada rangkaian seri. Untuk harga RLC yang sama, $Q_p = \frac{1}{Q_s}$ atau $Q_p = \frac{R_p}{X_p}$

Pada komponen RL



Untuk rangkaian parallel RL: $Q = \frac{R}{\omega_0 L}$

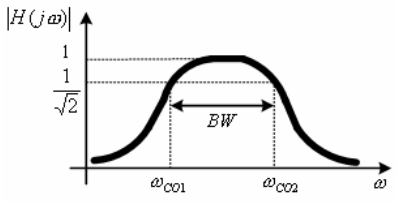
Pada rangkaian RLC



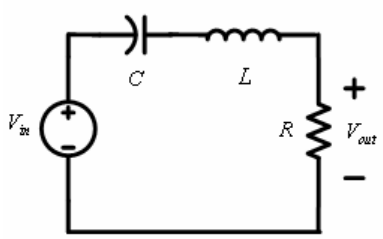
$$Q = \frac{R}{\omega_0 L} = \omega_0 RC$$

Bandwidth (BW) 3dB

Lebar pita pada saat terjadi level dayanya adalah $\frac{1}{2}$ dari daya maksimum



Perhatikan gambar rangkaian berikut:



Fungsi transfer rangkaian di atas adalah sebagai berikut:

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{1 + \frac{j(\omega L - 1)}{\omega C}} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)}$$

Jika rangkaian di atas mempunyai faktor kualitas rangkaian seri RLC dimana dinyatakan dengan:

$$Q = \frac{\omega_0 L}{R} \Rightarrow \frac{L}{R} = \frac{Q}{\omega_0}$$

$$Q = \frac{1}{\omega_0 CR} \Rightarrow \frac{1}{CR} = Q\omega_0$$

Maka fungsi transfer di atas dapat dinyatakan dengan persamaan:

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)} = \frac{1}{1 + j\left(\omega \frac{Q}{\omega_0} - \frac{1}{\omega} Q\omega_0\right)} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Respon frekuensi magnitudenya:

$$\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$

Saat level dayanya adalah setengah dari daya maksimum atau respon frekuensi magnitudenya sebesar $\frac{1}{\sqrt{2}}$, maka:

$$\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{1}{\sqrt{1 + Q^2 + \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2 = 1$$

$$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \frac{1}{Q}$$

Sehingga:

$$\omega^2 - \frac{\omega_0}{Q}\omega - {\omega_0}^2 = 0$$

Rumus….ABC:

$$\omega_{1,2} = \frac{\frac{\omega_0}{Q} \pm \sqrt{\frac{{\omega_0}^2}{Q^2} + 4{\omega_0}^2}}{2} = \frac{\omega_0}{2Q} \pm \frac{\omega_0}{2}\sqrt{\frac{1}{Q^2} + 4} = \frac{\omega_0}{2Q} \pm \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2}$$

Dimana: $\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} > \frac{\omega_0}{2Q}$, maka:

$$\omega_1 = \omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}$$

$$\omega_2 = \omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

Dari gambar respon frekuensi magnitude di atas didapat bahwa:

$$BW = \omega_{co2} - \omega_{co1} = \omega_2 - \omega_1$$

$$BW = \frac{\omega_0}{Q}$$

Atau:

$$\omega_1 = \omega_0 - \frac{BW}{2}$$

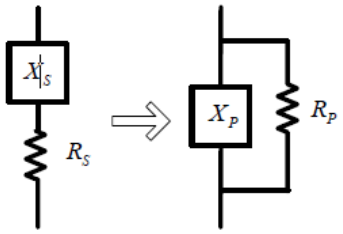
$$\omega_2 = \omega_0 + \frac{BW}{2}$$

Factor kualitas dapat dinyatakan sebagai perbandingan frekuensi resonansi terhadap bandwidth.

$$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

Frekuensi resonansi $f_0$adalah rata-rata geometri f_1 dan f_2:

<u>Konversi Faktor Kualitas Rangkaian Seri-Paralel</u>



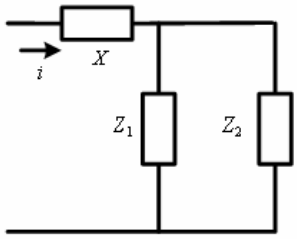
$$R_p = R_s(1+Q^2)$$

$$X_p = \frac{R_p}{Q} = \frac{R_s}{Q}(1+Q^2)$$

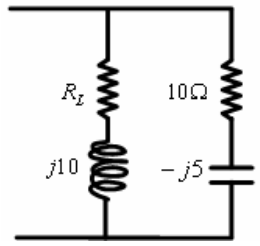
Soal untuk dikerjakan:

1. Jika V = 480V, $Z^1 = 12\angle 30^0$, $Z_2 = 12\angle -140^0$

a. Tentukan nilai komponen reaktif X pada saat resonansi $f_0 = 60$ Hz
b. Tentukan nilai i pada saat resonansi



2. Tentukakan komponen R_L agar terjadi resonansi!



3. Suatu rangkaian seri RLC dengan R = 50 Ω, L = 0,05 H, C = 20μF terpasang pada V =$100\angle 0^0$volt dengan frekuensi variabel. Pada frekuensi berapa tegangan inductor mencapai maksimum? Berapakah tegangan inductor tersebut?